AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST MEDICAL PLUS -01 TEST - 19 B SOLUTION Date 22-09-2019

[PHYSICS]

1. $t=\frac{A}{a}\sqrt{\frac{2}{g}}[\sqrt{H_1}-\sqrt{H_2}]$

Now, $T_1=\frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H}-\sqrt{\frac{H}{\eta}}\right]$

and $T_2=\frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{H}{\eta}}-\sqrt{0}\right]$

According to problem $T_1 = T_2$

$\therefore \quad \sqrt{H}-\sqrt{\frac{H}{\eta}}=\sqrt{\frac{H}{\eta}}-0 \Rightarrow \sqrt{H}=2\sqrt{\frac{H}{\eta}} \Rightarrow \eta=4$

2. Pressure at the bottom of tank $P = h\rho g = 3\times10^5\ \frac{N}{m^2}$.

Pressure due to liquid column $P_l = 3\times10^5 - 1\times10^5 = 2\times10^5$ and velocity of water $v=\sqrt{2gh}$

$\therefore \quad v=\sqrt{\frac{2P_l}{\rho}}=\sqrt{\frac{2\times2\times10^5}{10^3}}=\sqrt{400}$ m/s

3. Effective value of acceleration due to gravity becomes $(g+a_0)$.

4. $x=\sqrt{2gh_1}\times\sqrt{\frac{2h_2}{g}}$ or $x=2\sqrt{h_1h_2}$

Now, imagine a hole at a depth h_2 below the free surface of the liquid. The height of this hole will be h_1. Clearly, x remains the same.

5. $v=\sqrt{2gh}$

But $\quad p=h\rho g$ or $\frac{p}{\rho}=gh$

$\therefore \quad v\sqrt{\frac{2p}{\rho}}=\sqrt{\frac{2\times2\times10^5}{10^3}}\ \text{ms}^{-1}=20\ \text{ms}^{-1}$

$R^2v =$ constant

6. From Torricelli's theorem

$$v = \sqrt{2gd} \qquad \text{(i)}$$

where v is horizontal velocity and d is the depth of water in barrel.

Time t to hit the ground is given by

$$h = \frac{1}{2}gt^2 \quad \text{or} \quad t = \sqrt{\frac{2h}{g}}$$

$$\therefore \quad R = vt = \sqrt{(2gd)}\sqrt{\frac{2h}{g}} = 2\sqrt{dh} \quad \text{(Using (i))}$$

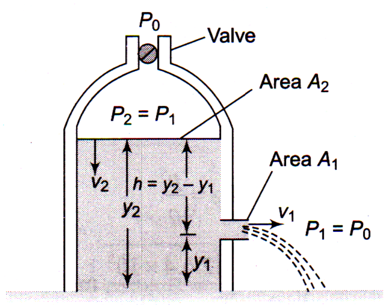
$$\therefore \quad R^2 = 4dh \text{ or } d = \frac{R^2}{4h}$$

7. $4(H-4) = 6(H-6)$

or $\quad 2H = 36 - 16 - 20$ or $H = 10$ cm

8. From equation of continuity, $v_2 = \frac{A_1}{A_2}v_1$

Since $A_1 << A_2$, v_2 must be very small compared to velocity of efflux at the hole, therefore we can take $v_2 = 0$.



Fluid emerging from the hole is open to atmospheric pressure P_0, We take two points A and B at the top of the fluid and at the hole respectively. From Bernoulli's principle,

$$P_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_1 + \rho g y_2$$

Solving for v_1, we obtain $v_1 = \sqrt{\frac{2(p_t - p_0)}{\rho} + 2gh}$

9. Velocity of water coming out from hole A

$$= v_1 = \sqrt{2gh}$$

Velocity of water coming out from hole B

$$= v_2 = \sqrt{2g(H-h)}$$

Time taken by water to reach the ground from hole A

$$= t_1 = \sqrt{2(H-h)/g}$$

Time taken by water to reach the ground from hole B

$$= t_2 = \sqrt{2h/g}$$

Obviously, range on the ground for both is the same

$\therefore \quad R = v_1 t_1 = v_2 t_2 = 2g\sqrt{h(H-h)}$

10. Let A and a be the cross-sectional areas of the vessel and hole respectively. Let h be the height of water in the vessel at time. Let $\left(-\frac{dh}{dt}\right)$ represent the rate of fall of level.

Then, $A\left(-\frac{dh}{dt}\right) = \alpha v = a\sqrt{2gh}$

or $\quad -\frac{dh}{\sqrt{h}} = \frac{\alpha\sqrt{2g}}{A}dt$

$$-\int_A^0 \frac{1}{\sqrt{h}}dh = \frac{a\sqrt{2g}}{A}\int_0^g dt$$

$$-(-2\sqrt{h}) = \frac{\alpha\sqrt{2g}}{A}t$$

or $\quad t = \frac{A}{\alpha}\frac{1}{\sqrt{2g}} \times 2\sqrt{h}$ or $t = \frac{A}{\alpha}\sqrt{\frac{2h}{g}}$

Now, $t \propto \sqrt{h}$

When h is quadrupled, t is doubled.

11. Let A = The area of cross section of the hole

v = Initial velocity of efflux

d = Density of water,

Initial volume of water flowing out per second = Av

Initial mass of water flowing out per second = Avd

Rate of change of momentum = Adv^2

Initial downward force on the flowing out water = Adv^2

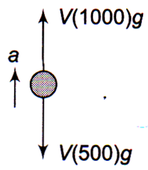
So equal amount of reaction acts upwards on the cylinder.

$\therefore$ Initial upward reaction = Adv^2 [As $v=\sqrt{2gh}$]

$\therefore$ Initial decrease in weight = $Ad(2gh)$

$$= 2Adgh = 2 \times \left(\frac{1}{4}\right) \times 1 \times 980 \times 25 = 12.5 \text{ gm-wt.}$$

12. Velocity of ball when it reaches to surface of liquid



$a = \dfrac{1000 \text{ gV} - 500 \text{ gV}}{500 \text{ V}}$; where V is the volume of the ball.

$a = 10 \text{ m/sec}^2$

Apply $v = u + at \Rightarrow 0 = \sqrt{2gh} - 10t$

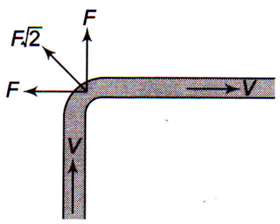
$\Rightarrow \quad \sqrt{2gh} = 10 \times (2)$

$\Rightarrow \quad 2 \times 10 \times h = 400 \Rightarrow = 20 \text{ m}$

13. Force exerted in vertical direction and horizontal direction are

$$F_1 = F_2 = v_{\text{rel}} \times \frac{dm}{dt} = V\rho \cdot L$$

$\Rightarrow \quad F_{\text{net}} = \rho VL\sqrt{2}$



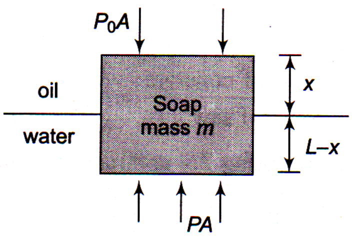
14. The velocity of system will not change in horizontal direction as water is leaking out vertically down. Because leaking water does not exchange any momentum with trolley in horizontal direction.

15. Tension in spring T = upthrust – weight of sphere

$= V\sigma g - V\rho g = V\eta\rho g - V\rho g \qquad (\text{As } \sigma = \eta\rho)$

$= (\eta - 1)V\rho g = (\eta - 1)mg$

16. Let A be area of soap bar.



$$PA - P_0A = mg \Rightarrow (P - P_0)A = mg$$

$\Rightarrow$ $[300gx + 1000g(L-x)A = AL\ 800g$

$\Rightarrow$ $\frac{x}{L} = \frac{2}{7}$

17. Let specific gravities of concrete and saw dust are ρ_1 and ρ_2 respectively.

According to principle of floatation weight of whole sphere = upthrust on the sphere

$$\frac{4}{3}\pi(R^3 - r^3)\rho_1 g + \frac{4}{3}\pi r^3 \rho_2 g = \frac{4}{3}\pi R^3 \times 1 \times g$$

$\Rightarrow$ $R^3\rho_1 - r^3\rho_1 + r^3\rho_2 = R^3$

$\Rightarrow$ $R^3(\rho_1 - 1) = r^3(\rho_1 - \rho_2) \Rightarrow \frac{R^3}{r^3} = \frac{\rho_1 - \rho_2}{\rho_1 - 1}$

$\Rightarrow$ $\frac{R^3 - r^3}{r^3} = \frac{\rho_1 - \rho_2 - \rho_1 + 1}{\rho_1 - 1}$

$\Rightarrow$ $\frac{(R^3 - r^3)\rho_1}{r^3\rho_2} = \left(\frac{1-\rho_2}{\rho_1 - 1}\right)\frac{\rho_1}{\rho_2}$

$\Rightarrow$ $\frac{\text{Mass of concrete}}{\text{Mass of saw dust}} = \left(\frac{1-0.3}{2.4-1}\right) \times \frac{2.4}{0.3} = 4$

18. Maas of liquid in horizontal portion of U-tube $= A\,d\rho$

Pseudo force on this mass $= Ad\rho a$

Force due to pressure difference in the two limbs

$= (h_1\rho g - h_2\rho g)A$

Equating, $(h_1 - h_2)\rho g A = Ad\rho a$

or $h_1 - h_2 = \frac{Ad\rho a}{\rho g A} = \frac{ad}{g}$

19. Let l be the length of the cylinder, when vertical, in water. Let A be the cross-sectional area of the cylinder. Equating weight of the cylinder with the upthrust, we get

$$Mg = Al\rho g \text{ or } m = Al\rho$$

When the cylinder is tilted through an angle θ, length of cylinder in water $= \dfrac{l}{\cos\theta}$

Weight of water displaced $= \dfrac{l}{\cos\theta} A\rho g$

Restoring force $= \dfrac{lA\rho g}{\cos\theta} - lA\rho g$

$$= lA\rho g\left[\frac{1}{\cos\theta} - 1\right] = mg\left[\frac{1}{\cos\theta} - 1\right]$$

20. Let m gwt be the weight of object in vacuum.

Volume of object $= \dfrac{m}{3.4}$

Weight of air displaced by object $= \dfrac{m}{3.4} \times 0.0012$

Volume of brass weight $= \dfrac{m}{8}$

Weight of air displaced by brass weights

$$= \frac{m}{8} \times 0.0012$$

Error = difference in buoyancy

$$= 0.0012\left[\frac{1}{3.4} - \frac{1}{8}\right]$$

Fractional error $= \dfrac{0.0012 \times 4.6}{3.4 \times 8} = 2 \times 10^{-4}$

21. Suppose volume and density of the body be V and ρ respectively, The, according to law of flatiron in water.

Weight = upthrust

$$V\rho g = \frac{2}{3} V\rho_w g \quad (1)$$

In liquid, $V\rho g = \dfrac{1}{4} V\rho_L g$ (2)

From (1) and (2), $\dfrac{2}{3} V\rho_w g = \dfrac{1}{4} V\rho_L g$

or $\dfrac{\rho_L}{\rho_w} = \dfrac{2/3}{1/4} = \dfrac{8}{3}$

or $\rho_L = \dfrac{8}{3}\rho_w = \dfrac{8}{3} \times 1$ g/cc

22. Velocity u of the body when it enters the liquid is given by $mgh = \frac{1}{2}mu^2$ or $u = \sqrt{2gh}$

Let Volume of the body = V

Mass of the body = Vd

Weight of the body = Vdg

Mass of liquid displaced = VD

Weight of liquid displaced = VDg

Net upward force = $VDg - VDg$

$= V_g(D - d)$

$$\text{Retardation} = \frac{\text{net weight}}{\text{mass}}$$

$$= \frac{V(D-d)g}{Vd} = \left(\frac{D-d}{d}\right)g$$

$$\text{Acceleration } a = -\left(\frac{D-d}{d}\right)g$$

Final velocity, v in the liquid when the body is instantaneously at rest is zero. Let the time taken be t.

$$v = u + at$$

$$0 = \sqrt{2gh} - \left(\frac{D-d}{d}\right)gt \cdot \left(\frac{D-d}{d}\right)gt = \sqrt{2gh}$$

$$\therefore \quad t = \left[\frac{d}{D-d}\right]\sqrt{\frac{2h}{g}}$$

23. The weight of the aircraft is balanced by the upward force due to the Pressure difference.

i.e.,

$$\Delta P = \frac{mg}{A} = \frac{(4 \times 10^5 \text{ kg})(10 \text{ ms}^{-2})}{500 \text{ m}^2} = \frac{4}{5} \times 10^4 \text{ N m}^{-2}$$

$$= 8 \times 10^3 \text{ N m}^{-2}$$

Let v_1, v_2 are the speed of air on the lower and upper surface of the wings of the aircraft and P_1, P_2 are the pressures there.

Using Bernoulli's theorem, we get

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}(\rho v_2^2 - \rho v_1^2)$$

$$\Delta P = \frac{\rho}{2}(v_2 + v_1)(v_2 - v_1)$$

$$\text{or} \quad v_2 - v_1 = \frac{\Delta P}{\rho v_{av}}$$

Here, $v_{av} = \frac{v_1 + v_2}{2} = 720 \text{ km h}^{-1}$

$$= 720 \times \frac{5}{18} \text{ms}^{-1} = 200 \text{ ms}^{-1}$$

$$\therefore \quad \frac{v_2 - v_1}{v_{av}} = \frac{\Delta P}{\rho v_{av}^2} = \frac{\frac{4}{5} \times 10^4}{1.2 \times (200)^2}$$

$$= \frac{4 \times 10^4}{5 \times 1.2 \times 4 \times 10^4} = 0.17$$

24. Total cross-sectional area of the femurs is,

$$A = 2 \times 10 \text{ cm}^2 = 2 \times 10 \times 10^{-4} \text{ m}^2 = 20 \times 10^{-4} \text{ m}^2$$

Force acting on them is

$$F = mg = 40 \text{ kg} \times 10 \text{ ms}^{-2} = 400 \text{ N}$$

$\therefore$ Average pressure sustained by them is

$$P = \frac{F}{A} = \frac{400 \text{ N}}{20 \times 10^{-4} \text{ m}^2} = 2 \times 10^5 \text{ N m}^{-2}$$

25. Since pressure is transmitted undiminished throughout the water

$$\therefore \quad \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

where F_1 and F_2 are the forces on the smaller and on the larger pistons respectively and A_1 and A_2 are the respective areas.

$$\therefore \quad F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi (D_2/2)^2}{\pi (D_1/2)^2} F_1 \left(\frac{D_2}{D_1}\right)^2 F_1$$

$$= \frac{(3 \times 10^{-2} \text{ m})^2}{(1 \times 10^{-2} \text{ m})^2} \times 10 \text{ N} = 90 \text{ N}$$

26. The Bernoulli's theorem is in one way the principle of conservation of energy for a flowing liquid (or gas). When an incompressible and non-viscous liquid (or gas) flows in stream-lined motion from one place to another, then at every point of its path the total energy per unit volume (pressure energy + kinetic energy + potential energy) is constant.

That is,

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

where P is pressure, ρ is density, g is gravity, v is velocity and h is height.

27. The scent sprayer is based on Bernoulli's theorem.

28. From Archemedes' principle, this apparent loss in weight is equal to the weight of the liquid displaced by the body.

Also, volume of candle = Area × length

$$= \pi\left(\frac{d}{2}\right)^2 \times 2L$$

Weight of candle = Weight of liquid displaced

$$V\rho g = V'\rho' g'$$

$$\Rightarrow \quad \left(\pi\frac{d^2}{4} \times 2L\right)\rho = \left(\pi\frac{d^2}{4} \times L\right)\rho'$$

$$\Rightarrow \quad \frac{\rho}{\rho'} = \frac{1}{2}$$

Since candle is burning at the rate of 2 cm/h, then after an hour, candle length is $2L - 2$

$$\therefore \quad (2L-2)\rho = (L-x)\rho'$$

$$\therefore \quad \frac{\rho}{\rho'} = \frac{L-x}{2(L-1)}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{L-x}{2(L-1)}$$

$$\Rightarrow \quad x = 1 \text{ cm}$$

Hence, in one hour it melts 1 cm and so it falls at the rate of 1 cm/h.

29. According to Bernoulli's principle

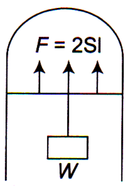
$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

At the sides the velocity is higher, so the pressure is lower. But the pressure at a given horizontal level must be equal, therefore the liquid rises at the sides to some height to compensate for this drop in pressure.

30. Because film tries to cover minimum surface area.

31. Here, $W = 1.5 \times 10^{-2}$ N,

$$l = 30 \text{ cm} = 30 \times 10^{-2} \text{ m}$$



A liquid film has two free surfaces. A slider will support the weight when the force of surface tension action upwards on the slider ($2Sl$) balances the downward force due to weight (= W)

32. Energy needed = Increment in surface energy

= (surface energy of n small drops) – (surface energy of one big drop)

$= n4\pi r^2 T - 4\pi R^2 T = 4\pi T(nr^2 - R^2)$

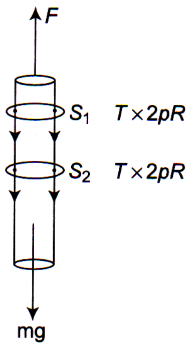
33. $W = 8\pi T(r_2^2 - r_1^2) = 8\pi T\left[\left(\frac{2}{\sqrt{\pi}}\right)^2 - \left(\frac{1}{\sqrt{\pi}}\right)^2\right]$

$\therefore \quad W = 8 \times \pi \times 30 \times \frac{3}{\pi} = 720$ erg

34. The force exerted by film on wire or thread depends only on the nature of material of the film and not on its surface area. Hence the radius of circle formed by elastic thread does not change.

35. The free body diagram of the capillary tube is as shown in the figure. Net force F required to hold tube is

F = force due to surface tension at cross-section



Free body daigram of capillary tube

$(S_1 + S_2)$ + weight of tube.

$= (2\pi RT + 2\pi RT) + mg = 4\pi RT + mg$

36. The thin ring is in contact with water from both inside and outside. So, contact length is $2 \times 20 = 40$ cm

$F_{\min} = F_{ST} + W = (75 \times 10^{-5}) \times 40 + 0.1 = 0.130$ N

37. It may be noted that the soap film has two free surfaces. So, the effective length is 8ℓ.

38. Effective area $= 2 \times 0.02\ m^2 = 0.04\ m^2$

Surface energy $= 5\ m^{-1} \times 0.04\ m^2 = 2 \times 10^{-1}$ J

39. $W = [2 \times 4\pi(3r)^2 - 2 \times 4\pi r^2]\ T = 64\ \pi r^2 T$

40. $F = 2\pi r_1 T + F = 2\pi r_2 T$

$= 2\pi(r_1 + r_2)T$

$= 2 \times 3.14(10 + 5)(72) = 6782.4$ dyne

41. $h = \frac{2\sigma \cos\theta}{r\rho g}$ or $r = \frac{2\sigma \cos\theta}{h\rho g}$

or $r = \frac{2 \times 75 \times 10^{-3} \times \cos 0°}{3 \times 10^{-2} \times 10^{3} \times 10} \text{m} = 5 \times 10^{-4}$ m

42. $h = h_0 = \frac{2T \cos\theta}{\rho g r}$

$= \frac{2(72)\cos 0°}{(1)(1000)\left(\frac{1}{40}\right)} = 57.6$ cm

Since ℓ (= 50 cm) < h_0.

$h = 50$ cm

43. Excess pressure inside the air bubble $= \frac{2T}{r}$

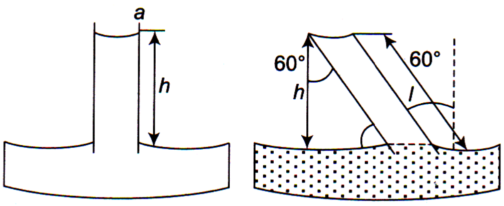
$\Rightarrow \quad P_{in} - P_{out} = \frac{2T}{r} = \frac{2 \times 70 \times 10^{-3}}{0.1 \times 10^{-3}} = 1400$ Pa

$\Rightarrow \quad P_{in} = 1400 + 1.013 \times 10^5$

$= 0.014 \times 10^5 + 1.013 \times 10^5 = 1.027 \times 10^5$ Pa

44. $h = \frac{2T \cos\theta}{rdg} \therefore h \propto \frac{1}{r}$. So the graph between h and r will be rectangular hyperbola.

45. Since water rises to height of 2 cm in a capillary



If tube is at 60°. In this case height must be equal to

$h = 2$ cm

$\Rightarrow \quad \cos 60° = \frac{h}{l}$

$\therefore \quad l = \frac{h}{\cos 60°} = \frac{2}{1/2} = 4$ cm